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We propose an experiment for entangling two spatially separated Bose-Einstein condensates by Bragg scattering of light. When Bragg scattering in two condensates is stimulated by a common probe, the resulting quasiparticles in the two condensates get entangled due to quantum communication between the condensates via probe beam. The entanglement is shown to be significant and occurs in both number and quadrature phase variables. We present two methods of detecting the generated entanglement.

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Inseparability of quantum states of two or more subsystems is the most significant feature of quantum mechanics. Apparently puzzling, yet most profound, first formulated as a paradox [1], this inseparability known as quantum entanglement lies at the very heart of non-classical physics. Further, as a basic resource for quantum information processing, it has become a focal theme of research in modern physics and many issues in the foundations of quantum mechanics. Generation and manipulation of entanglement is, therefore, of prime interest. Bose-Einstein condensates (BEC) [2] of weakly interacting atomic gases seem to be suitable macroscopic objects for producing many-particle entanglement [3]. A BEC has intrinsic entanglement character due to reduced quantum fluctuations in momentum space. For instance, in the condensate ground state, a pair of mutually opposite momentum modes is maximally entangled in atomic number variables [4].

Stimulated resonant Bragg scattering of light by a condensate generates quasiparticles [5], predominantly in two momentum side-modes \mathbf{q} and $-\mathbf{q}$, where \mathbf{q} is the momentum transferred from light fields to the atoms. Momentum side-modes are the excited states of a BEC, atoms in such a state collectively behave as quasiparticles. Bragg spectroscopy [6] with coherent or classical light produces coherent states of the quasiparticles in a BEC. When these quasiparticles are projected into particle domain, that is, into the Bogoliubov-transformed momentum modes [7], they form two-mode squeezed as well as entangled state [4]. Bragg spectroscopy with non-classical light can generate tripartite entanglement [4] in a condensate. In addition to atom number and phase variables, spin degree-of-freedom of a spinor BEC [8] can be useful in describing entanglement in spin variables. Thus, BECs offer a fertile ground for studying different aspects of entanglement. Apart from BECs, multi-atom

entanglement in other macroscopic systems has been realized [9] on the basis of collective spin squeezing [10,11]. Further the entanglement in collective spin variables of two ensembles of gaseous Cs atoms has been experimentally demonstrated [12]. Continuous variables like the quadratures of a field mode (which are analogous to position and momentum) have also been employed [13] in entanglement studies.

We here propose a scheme for producing quantum entanglement between two spatially separated BECs of a weakly interacting atomic gas. The entanglement we consider is in quasiparticles of BECs. The proposed experiment is schematically shown in Fig.1. The condensates A and B are illuminated by pump lasers L1 and L2, respectively. A single stimulating probe laser L3 passes through both the condensates. All these three lasers are detuned far off the resonance of an electronic excited state of the atoms. The frequencies and the directions of propagation of these lasers are so chosen such that Bragg resonance (phase matching) conditions of scattering in both the condensates are fulfilled. The Hamiltonian of the system is $H = H_A + H_B + H_F + H_{AF} + H_{BF}$. Retaining the dominant momentum side-modes \mathbf{q} and $-\mathbf{q}$ only under Bragg resonance condition, in the Bogoliubov approximation [7], $H_A = \hbar\omega_q^B (\hat{\alpha}_q^\dagger \hat{\alpha}_q + \hat{\alpha}_{-q}^\dagger \hat{\alpha}_{-q})$, where $\hat{\alpha}_q$ represents quasi-particle with momentum \mathbf{q} , and $\omega_q^B = [(\omega_q + \frac{\mu}{\hbar})^2 - (\frac{\mu}{\hbar})^2]^{1/2}$ is the frequency of Bogoliubov's quasi-particle [7]. Here $\omega_q = \frac{\hbar^2 q^2}{2m}$, and $\mu = \frac{\hbar^2 \xi^{-2}}{2m}$ is the chemical potential with $\xi = (8\pi n_0 a_s)^{-1/2}$ being the healing length. Similarly, $H_B = \hbar\omega_q^B (\hat{\beta}_q^\dagger \hat{\beta}_q + \hat{\beta}_{-q}^\dagger \hat{\beta}_{-q})$,

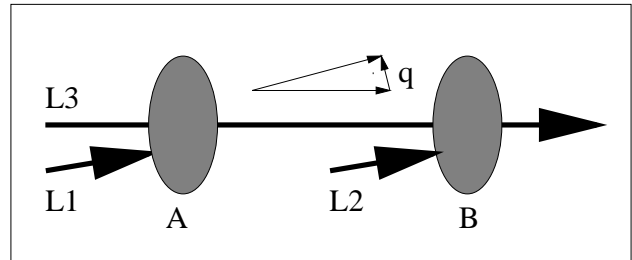


FIG. 1. The scheme for creation of entanglement. A and B are two condensates, L1 and L2 are pump lasers, L3 is a common entangling probe laser. Both the pumps have same wave vector \mathbf{k}_1 , probe's wave vector is \mathbf{k}_2 . The probe is red-detuned from the pumps. The lasers are in Bragg resonance with a particular momentum mode \mathbf{q} of both the condensates.

with $\hat{\beta}_q$ being the quasi-particle operator of the con-

condensate B. The pumps are treated classically. Let \hat{c} represents the common probe field mode, then the field Hamiltonian $H_F = -\hbar\delta\hat{c}^\dagger\hat{c}$, where $\delta = \omega_1 - \omega_2$ is the pump-probe detuning. The quasiparticle operators $\hat{\alpha}(\hat{\beta})$ are related to the particle operators $\hat{a}(\hat{b})$ by Bogoliubov's transformation: $\hat{a}_{\mathbf{q}} = u_{\mathbf{q}}\hat{\alpha}_{\mathbf{q}} - v_{\mathbf{q}}\hat{\alpha}_{-\mathbf{q}}^\dagger$, where $v_{\mathbf{q}} = (u_{\mathbf{q}}^2 - 1)^{1/2} = [\frac{1}{2}(\frac{\omega_{\mathbf{q}} + \mu/\hbar}{\omega_{\mathbf{q}}^B} - 1)]^{1/2}$. The atom-field interaction Hamiltonian for condensate A is

$$H_{AF} = \hbar\eta_A\hat{c}^\dagger(\hat{\alpha}_{\mathbf{q}}^\dagger + \hat{\alpha}_{-\mathbf{q}}) + \text{H.c.} \quad (1)$$

where $\eta_A = \sqrt{N_A}\Omega_A f_q$ is the effective atom-field coupling constant. Here N_A is the number of atoms in condensate A, Ω_A is the two-photon Rabi frequency of an atom in A and $f_q = u_q - v_q$. H_{BF} is given by the similar expression as H_{AF} with subscript A replaced by B and α replaced by β . The Heisenberg equations of motion are

$$\dot{\hat{\alpha}}_{\mathbf{q}} = -i\omega_q\hat{\alpha}_{\mathbf{q}} - i\eta_A\hat{c}^\dagger \quad (2)$$

$$\dot{\hat{\alpha}}_{-\mathbf{q}}^\dagger = i\omega_q\hat{\alpha}_{-\mathbf{q}}^\dagger + i\eta_A\hat{c}^\dagger \quad (3)$$

$$\dot{\hat{c}}^\dagger = -i\delta\hat{c}^\dagger + i[\eta_A(\hat{\alpha}_{\mathbf{q}} + \hat{\alpha}_{-\mathbf{q}}^\dagger) + \eta_B(\hat{\beta}_{\mathbf{q}} + \hat{\beta}_{-\mathbf{q}}^\dagger)] \quad (4)$$

The Heisenberg equations of $\hat{\beta}_{\mathbf{q}}$ and $\hat{\beta}_{-\mathbf{q}}^\dagger$ are similar to those of $\hat{\alpha}$, but $\hat{\alpha}$ and η_A should be replaced by $\hat{\beta}$ and η_B , respectively.

We next discuss how to quantify entanglement between two BECs. If the entanglement occurs in number operators of the quasiparticle modes 1 and 2, then it can be quantified by the parameter [10,4]

$$\xi_n(1,2) = \langle [\Delta(\hat{n}_1 - \hat{n}_2)]^2 \rangle / (\langle \hat{n}_1 \rangle + \langle \hat{n}_2 \rangle). \quad (5)$$

If $\xi_n < 1$, then the two modes are entangled. If the entanglement is described by two noncommuting Gaussian operators \hat{X} and \hat{P} which are analogous to position and momentum variables, then the entanglement parameter is defined by [14]

$$\xi_p(1,2) = \frac{1}{2}[\langle [\Delta(X_1 + X_2)]^2 \rangle + \langle [\Delta(P_1 - P_2)]^2 \rangle] \quad (6)$$

The two modes are entangled in quadrature phase, when $\xi_p < 1$.

For numerical illustration, we consider two homogeneous identical Na condensates. We here enlist the important results: 1) If the modes \mathbf{q}_1 of A and \mathbf{q}_2 of B are in Bragg-resonance with the respective Bragg pulses, and if the effective coupling of B (η_B) is stronger than that of A, then entanglement arises between \mathbf{q}_1 of A and $-\mathbf{q}_2$ of B only, other pairs of modes are immune to any entanglement. In fig.2, we display entanglement parameters between these two chosen modes as a function of time. We set $\mathbf{q}_1 = \mathbf{q}_2 = \mathbf{q}$. The effective coupling can be

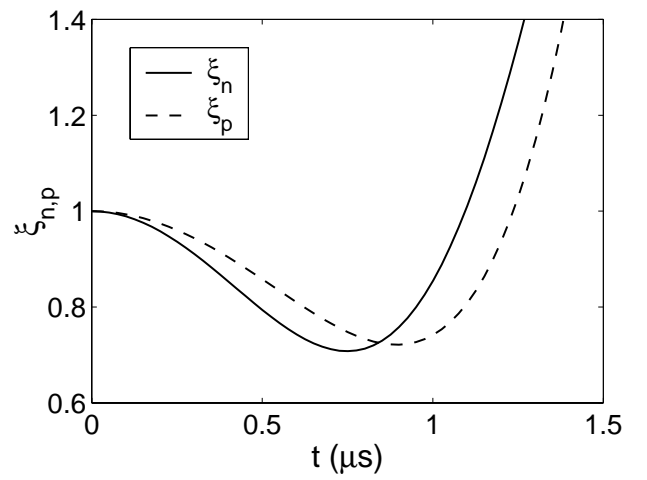


FIG. 2. Entanglement parameters ξ_n and ξ_p between \mathbf{q} -mode of A and $-\mathbf{q}$ -mode of B as a function of time in μs . For both the condensates, $\omega_q^B = 0.21$ MHz, $q = 2\xi^{-1}$ and $\delta = 0.17$ MHz. The coupling constants $\eta_A = 1.62$ MHz, $\eta_B = 1.25\eta_1$. Both the condensates are initially in the ground states (vacuum of quasi-particle operators), and the common probe field is in coherent state with average number of photons equal to 10.

made different either by using pump lights of different intensities or taking different atom numbers for the two otherwise identical condensates. 2) For equal couplings, there is no entanglement between any pair of modes. Fig.3 shows the variation of entanglement parameters as a function of the ratio of the two coupling constants at a fixed time. 3) We find entanglement both in quasiparticle (phonon) modes ($\hat{\alpha}, \hat{\beta}$), and in the Bogoliubov transformed modes of quasiparticles which we call particle or atomic modes (\hat{a}, \hat{b}). However, in atomic modes, entanglement is weaker than that in quasiparticle modes. It is worth mentioning that in a single condensate, as shown in Ref. [4], coherent light scattering can generate entanglement only in atomic modes, and not in phonon modes. In contrast, one can generate *entanglement in phonon modes in two condensates* by coherent light scattering. The light scattering events occurring at A and B are not independent, since a quantum communication has been set between the generated quasiparticles in A and B via the common probe. Had we treated the common *probe classically*, then the Hamiltonian (Eq.(1)) would have been linear in the atomic operators. A Hamiltonian linear in Bosonic operators can not generate nonclassical correlation. Therefore, the *probe* must be treated *quantum mechanically*. The probe carries with it quantum fluctuations of one condensate and transfers a part of it to the other leading to the entanglement between the two condensates.

To explain the results further, we here resort to an approximate analysis. Let us suppose, $\omega_q \ll \eta_{A(B)}$ and $\delta \ll \eta_{A(B)}$, then we can neglect the diagonal terms proportional to ω_q and δ in the Hamiltonian. For equal

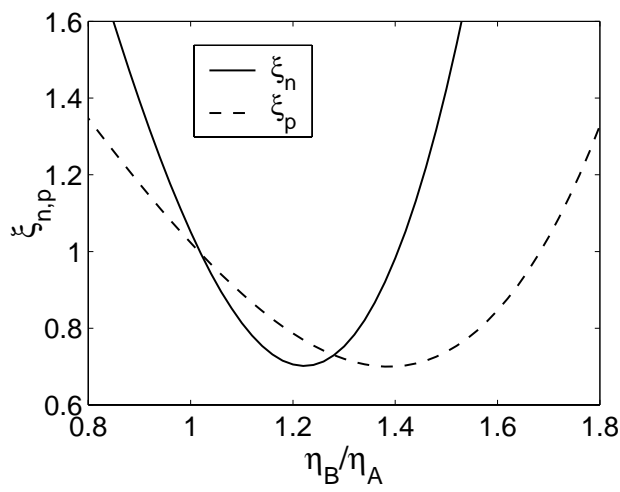


FIG. 3. The entanglement parameters ξ_n (solid) and ξ_p (dashed) as a function of η_B/η_A for a fixed time $t = 0.75 \mu s$. The other parameters are the same as in Fig.2.

coupling ($\eta_A = \eta_B$), from Heisenberg equation of motions, it then follows that $\hat{\alpha}_{\mathbf{q}}(t) + \hat{\beta}_{-\mathbf{q}}^\dagger(t) = \hat{\alpha}_{\mathbf{q}}(0) + \hat{\beta}_{-\mathbf{q}}^\dagger(0)$, that is, the superposition operator $\Sigma = \hat{\alpha}_{\mathbf{q}} + \hat{\beta}_{-\mathbf{q}}^\dagger$ becomes a constant of motion. Let us write the quadratures $X_A = \frac{1}{\sqrt{2}}(\hat{\alpha}_{\mathbf{q}} + \hat{\alpha}_{\mathbf{q}}^\dagger)$, $P_A = -i\frac{1}{\sqrt{2}}(\hat{\alpha}_{\mathbf{q}} - \hat{\alpha}_{\mathbf{q}}^\dagger)$, and similarly for X_B and P_B . Then it can be shown that $\xi_p = \frac{1}{4}[\langle[\Delta(\Sigma + \Sigma^\dagger)]^2\rangle - \langle[\Delta(\Sigma - \Sigma^\dagger)]^2\rangle]$. For equal coupling and the initial states being in vacuum or in coherent states, one obtains $\xi_p = 1$, that is, the two modes are unentangled. Let us then consider the case of different couplings, for short times characterized by $\eta_A t \ll 1$, and $\eta_B t \ll 1$, we obtain perturbative solutions of $\hat{\alpha}_{\mathbf{q}}(t)$ and $\hat{\beta}_{-\mathbf{q}}^\dagger(t)$ upto the second order in time. Using these solutions, we calculate $\xi_p = 1 - \eta_A \eta_B t^2 (1 - \frac{\eta_A}{\eta_B})$, which is less than unity (the two modes are entangled in quadrature variables) if $\eta_A \eta_B t^2 (1 - \frac{\eta_A}{\eta_B}) > 0$ which is only possible if $\eta_A \neq \eta_B$ and $\eta_A < \eta_B$. Similarly, we can prove that for $-\mathbf{q}$ (off-resonant) of A and \mathbf{q} (resonant) of B, $\xi_p = 1 + (\eta_B t)^2 (1 - \frac{\eta_A}{\eta_B})$ which is always greater than unity for $\eta_A < \eta_B$. For the same resonant \mathbf{q} -mode of A and B, $\xi_p = 1 + \eta_B t^2 / 2 + (\eta_A^2 + \eta_B^2) t^4 / 4$, which is always greater than unity. In the same way, we can show that, for the remaining mode-pair $(-\mathbf{q}, -\mathbf{q})$, ξ_p is also greater than unity.

Next, we prove that, to generate entanglement in number variables (ξ_n), the two coupling parameters should also be different. Substituting $\hat{n}_1 = \hat{\alpha}_{\mathbf{q}}^\dagger \hat{\alpha}_{\mathbf{q}}$ and $\hat{n}_2 = \hat{\beta}_{-\mathbf{q}}^\dagger \hat{\beta}_{-\mathbf{q}}$ in Eq.(5) and using the perturbative solutions we can express $\xi_n = 1 - R / (\langle \hat{n}_1 \rangle + \langle \hat{n}_2 \rangle)$ where

$$R = 8\eta_A^2 t^4 [\eta_B^2 - 2\eta_A^2 + 4n_p(\eta_B^2 - \eta_A^2)] \quad (7)$$

where n_p is the initial number of photons in the coherent probe beam. Now, $\xi_n < 1$ implies that $R > 0$ which amounts to $(\eta_B/\eta_A)^2 > 1 + 1/(1 + 4n_p)$, that is, $\eta_B > \eta_A$. On the other hand, if $\eta_B \leq \eta_A$, $\xi_n > 1$. We

also carry out an alternative analysis to check whether the two resonant modes \mathbf{q} of A and B exhibit any entanglement in other parameter regimes. By neglecting the off-resonant mode $-\mathbf{q}$ in both the condensates and keeping only the resonant mode, it can be analytically proved that $\xi_p(\mathbf{q}, \mathbf{q}) = 1 + \sinh^2(\eta t)$ and $\xi_n(\mathbf{q}, \mathbf{q}) = 1 + (1 + \frac{n_p}{n_p+1}) \frac{(\eta_A^2 - \eta_B^2)^2}{(\eta_A^2 + \eta_B^2)^2} \sinh^2(\eta t)$, that is, both the parameters ξ_p and ξ_n are always greater than unity. Here $\eta = \sqrt{\eta_A^2 + \eta_B^2}$.

We next show how a set up as shown in Fig.4 can be utilized to verify the generated entanglement. After the process of generation of entanglement, the duration of which can be typically on the order of 1 to 100 μs , is over, the lasers L1, L2 and L3 are switched off. Two different pairs of verifying pump-probe Bragg pulses are applied to the condensates, as described in the caption of Fig.4. The two probes should be derived from a common source. The modes \mathbf{q} of A and $-\mathbf{q}$ of B are in Bragg resonance with the respective Bragg pulses. The effective field-condensate couplings for both the condensates are very small compared to the Bogoliubov frequency ω_q^B . Let $\hat{c}_{probe,A}$ and $\hat{c}_{probe,B}$ denote the verifying probe field modes for the condensates A and B, respectively. By neglecting the off-resonant terms $\hat{\alpha}_{2\mathbf{q}}^\dagger \hat{\alpha}_{\mathbf{q}}$ and $\hat{\alpha}_{-2\mathbf{q}}^\dagger \hat{\alpha}_{-\mathbf{q}}$ in the Hamiltonian, the time evolution of the output probe modes, in a frame rotating with pump-probe detuning δ , can be written as

$$\begin{aligned} \hat{c}_{probe,A}^{(out)} &\simeq \hat{c}_{probe,A}^{(in)} + \frac{\eta_A}{\delta - \omega_q^B} (\exp[i(\delta - \omega_q^B)t] - 1) \hat{\alpha}_{\mathbf{q}}^\dagger \\ &\quad + \frac{\eta_1}{\delta + \omega_q^B} (\exp[i(\delta + \omega_q^B)t] - 1) \hat{\alpha}_{-\mathbf{q}} \end{aligned}$$

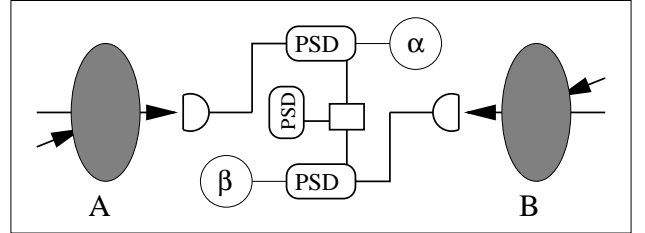


FIG. 4. The scheme for verification of the entanglement. Apart from pump lasers, two extremely weak verifying probes of the same frequency as that of entangling probe - one each for each condensate - are switched on. The probes acting on A and B have momentum \mathbf{k}_2 and $-\mathbf{k}_2$, respectively. The momentum of pump laser for A is \mathbf{k}_1 , while that for B is $-\mathbf{k}_1$. Thus, the mode \mathbf{q} and $-\mathbf{q}$ are in Bragg resonance with A and B, respectively. The output from the two phase sensitive detectors (PSD) can be integrated by an integrator followed by another phase-sensitive detection of the integrated signal.

$\hat{c}_{probe,B}^{(out)}$ is given by similar expression as above with α replaced by β ; and the subscripts A and \mathbf{q} replaced B and $-\mathbf{q}$, respectively. Thus the output probes have oscillating parts at frequency $\delta - \omega_q^B$ proportional to the quasiparti-

cle amplitudes $\alpha_{\mathbf{q}}$ and $\beta_{-\mathbf{q}}$, and at frequency $\delta + \omega_q^B$ proportional to the amplitudes $\alpha_{-\mathbf{q}}$ and $\beta_{\mathbf{q}}$. Therefore, phase-sensitive measurements of the spectral components of the output probe beams corresponding to these frequencies would provide measures of the quasiparticle operators. The output from both the PSDs can be integrated by an integrator and the integrated signal can also be measured by another PSD. By repeating the same measurements under identical conditions we could calculate the number variances or correlation functions of interest, which can be employed to calculate the entanglement parameter in number operators, i.e., ξ_n . For calculating entanglement parameters in quadrature phase variables, both the output probe beams coming from A and B, can be mixed via a beam splitter to form the superposition operators Σ which can be measured by a similar phase sensitive detection scheme (not shown in Fig.4).

Following the recent experiment of Ketterle's group [15], we also suggest that the quasiparticles can be detected by imparting a large momentum to them with additional Bragg pulses. Alternatively, in the large momentum regime ($q \gg \xi^{-1}$), the Bragg-scattered atoms which essentially behave as free particles ($\omega_q^B \propto q^2$), can be outcoupled by switching off the trap. Since entanglement is between two opposite momentum states, by proper geometric arrangement, the two moving entangled atomic ensembles can be made to collide and interfere. From the interference pattern obtainable via absorption imaging, the atomic number fluctuations can be deduced using the theoretical model used in Ref. [16], and thus entanglement parameter in number variables can be calculated.

In conclusion, we have theoretically demonstrated how light scattering leads to quantum entanglement between two Bose-Einstein condensates. We find that the quasiparticle or phonon as well as free-particle momentum modes of two condensates can be entangled. The quasiparticle state can be sufficiently long-lived due to weak nature of interatomic interaction and the constraints imposed by momentum conservations. The generated entanglement may be useful in quantum communication using coherent light [17]. We have particularly focused on the conditions under which the entanglement can be obtained. We have also suggested how quasiparticles could be studied by using Bragg scattering of far off resonant fields.

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